ters are young (3–5 Myr) and contain Wolf-Rayet stars. Consequently, this galaxy provides an excellent laboratory for studying young starbursts. The *ROSAT HRI* has been used to image the central regions of NGC 5253, finding a very complex morphology. The best fit model to the X-ray emission involved five point sources. The brightest of the X-ray point sources is centred on the brightest star clusters. However, some peaks in the X-ray emission are not associated with bright star clusters. The most plausible origin for the X-ray emission is that we are seeing the formation of small X-ray emitting superbubbles in the core of NGC 5253. Several unresolved questions remain as to their future evolution. Will these superbubbles merge to give rise to one central superbubble, that will give rise to a superwind such as is seen in M82? Or will the individual superbubbles 'blow-out' of the galaxy without merging, meaning a superwind will not form, and that NGC 5253 will retain much of its ISM? — JENNIFER HATCHELL.

THE DANJON LIMIT OF FIRST VISIBILITY OF THE LUNAR CRESCENT

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When the distinguished French astronomer André Danjon was the director of Strasbourg Observatory, he became engaged in determining the light curve of the Moon. In 1931 he noticed that the Moon of August 13, which was only 16·2 hr before new, extended only 75–80° from cusp to cusp. In other words, Danjon found that the outer terminator of the crescent was considerably less than a complete half-circle, which it should have been theoretically. This was not an isolated observation because other observations, and also examination of previous records, showed that this shortening of the crescent was a general and real phenomenon. Danjon also noticed that the shortening diminishes as the angular distance of the Moon from the Sun increases[1,2].

Danjon illustrated this phenomenon in a diagram[1] (his Fig. 29) which we have reproduced in Fig. 1, and which he explained as follows (ref. 1, p. 60):

> Let us represent the Moon ... by its projection on a plane passing through its center and those of the Earth and Sun. Light coming from the direction SO illuminates the left half of the globe, limited by the terminator BD. Since the Earth is in the direction OE, the hemisphere turned toward us is bounded by the great circle that projects as AC. On a smooth sphere, the zone AOB would appear sunlit, forming a 180-degree-long crescent with one cusp at O, the other at the diametrically opposite point of the sphere.
>
> But the Moon is not smooth, and the mechanism described above displaces the cusp from O to Q. The lunar surface in the little triangle OPQ remains invisible. We call PQ the *deficiency arc*, and evaluate it as follows. If a is the angular distance of the Moon from the Sun (taking account of lunar parallax), 2ω the length of the crescent (which would be 180° on a smooth sphere), the deficiency arc α is given by the formula $\sin\alpha = \sin a \cos\omega$.

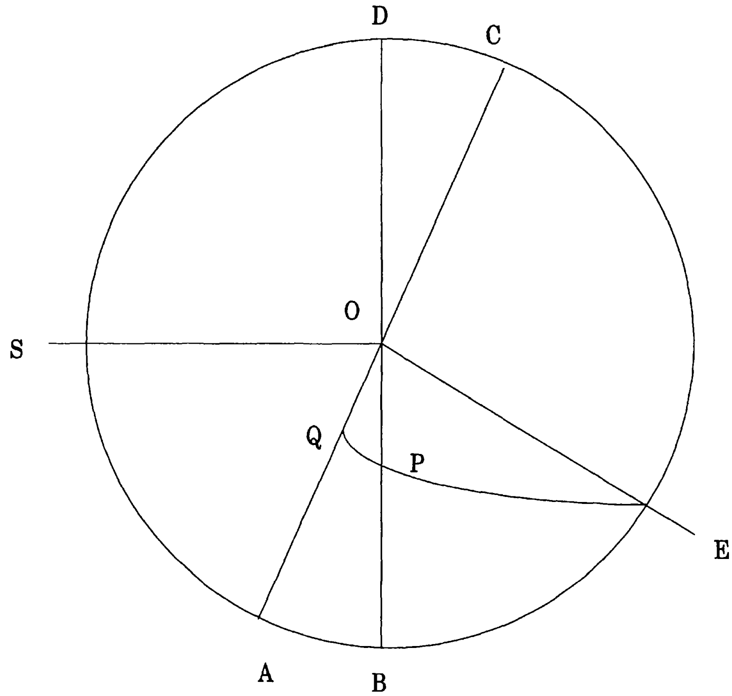

FIG. 1
The diagram of Danjon.

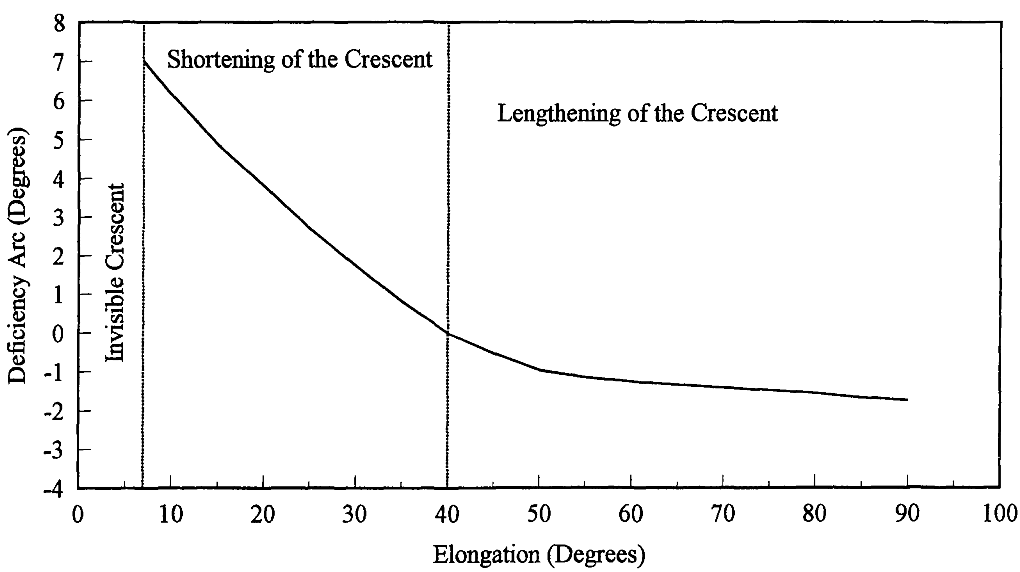

FIG. 2
Danjon limit.

Danjon collected 75 measurements and estimates of crescent length and calculated the deficiency arc (the amount of contraction of the sunlit crescent) in each case as a function of the elongation (taking account of lunar parallax). The result was as shown in Fig. 2 (the continuous line), which is a reproduction of Fig. 29 of Danjon's paper[3] — itself an improvement of Fig. 31 of a previous paper[1]. We have introduced the dotted lines and comments for illustration. The graph shows that when the Moon is exactly 7° from the Sun, then the deficiency arc is also 7°. This means, according to the above equation, that at this elongation the lunar crescent cannot be seen because no part of it will be sunlit. Obviously, when the elongation is less than 7° there would also be no sunlit crescent visible. However, the arc of deficiency decreases gradually with the increase in elongation until it starts to be negative after 40° of elongation, though the change of the deficiency arc with the elongation in this region is relatively slow. This means that when the Moon is between 7° and 40° from the Sun, the bright cusps of the crescent extend for less than 180°, *i.e.*, the effect is shortening of the crescent; whereas when the Moon is more than 40° from the Sun, its outer terminator extends for slightly more than a semicircle, *i.e.*, the effect would be lengthening of the crescent.

The phenomenon observed by Danjon has important implications for the determination of the first visibility of the lunar crescent. It indicates that, no matter what its age is, the crescent cannot be seen if it is less than 7° from the Sun, regardless of any favourable observing circumstances that may exist. The Moon of a specific age can be of different elongations from the Sun, depending on its latitude and whether near perigee or apogee. Danjon also noted that since the new Moon cannot pass more than $\simeq 5 \cdot 5°$ north or south of the Sun, which is less than the 7° limit, then the lunar crescent must disappear for a period of time during every lunation.

Danjon suggested that the phenomenon he observed is caused by the shadows of the lunar mountains. However, this explanation has been contested more recently by other researchers. McNally[4] disagreed with the explanation of Danjon, and also objected to an alternative explanation that attributes the phenomenon to distortions of the figure of the Moon. McNally stressed that modern measurements have shown that variations in height of the lunar mountainous terrain and departures from true sphericity are less than 0·6% of the lunar radius; so the arc of shortening cannot be caused by distortions of the Moon's shape from sphericity. McNally has explained the Danjon limit in terms of atmospheric seeing (turbulence). He suggested that the atmospheric seeing causes the crescent to be invisible where the cusp is thinner than the size of the "seeing disc". He thought that if the angular size of the seeing disc is larger than the width of the crescent, then the illumination of the crescent will be spread over a wider area and the illumination for unit area will thereby be reduced. The contrast between crescent and sky having been reduced by the seeing therefore renders the thinner part of the crescent more difficult to detect for both the naked eye and telescope. While conceding that variables such as atmospheric clarity, contrast with sky background, and visual response do all modify how the lunar crescent is actually observed by the human eye or instrument, McNally emphasized that these latter factors are secondary to the effect of the atmospheric seeing. McNally also suggested that, according to his model, the Danjon limit should be 5° rather than 7° as indicated by Danjon[4]. Finally, he conceded that while his model can explain the shortening in the length of the lunar crescent it does not explain why the outer terminator of the Moon can lengthen at large elongations to greater than 180°.

Schaefer also investigated the phenomenon of Danjon's limit. Though he first accepted (ref. 5, p. 35) the interpretation of Danjon, he later rejected attributing the phenomenon to shadows of lunar mountains because the required shadow length would have to be a function of the Earth's position and because mountain chains would have to average over 12 000 metres in height[6]. Schaefer also gave three reasons for disagreeing with McNally's explanation. Firstly, physiological experiments show that the visibility of unresolved sources does not depend on the smearing imposed on the source. Secondly, the resolution of the human eye (42″ or larger) is always much larger than the size of the seeing disc so that seeing has no appreciable effect on the perceived width — and hence visibility — of the cusp. Thirdly, telescopic and visual observers report essentially the same arc shortening although the perceived cusp widths differ by large amounts[7].

Schaefer has pointed out that "for naked eye observations, the critical portions of the crescent are always narrower than the resolution of the eye. In such a case, the detection threshold does not depend on the surface brightness of the Moon, but on the total brightness integrated across the crescent" (ref. 6, p. 271). He therefore suggests that the shortening of the length of the lunar crescent is due to sharp falling off of the brightness integrated across the crescent towards the cusps. This he attributes to three reasons, the first of which is that the crescent gets rapidly narrower. The second reason is that the cusps are regions illuminated by the Sun when very close to the local horizon and hence, on average, the polar terrain is illuminated less than equatorial terrain because of greater foreshortening. Thirdly, he suggested that the macroscopic roughness of the surface of the Moon creates shadows at the lunar poles that cover more of the illuminated surface than at the lunar equator[6]. On the other hand, Schaefer does confirm that the Danjon limit is 7°.

In a recent private communication with the authors, Bernard Yallop raised the following point: "Is there another explanation: that when Earthshine on the main disc of the Moon matches the brightness of the crescent, which it will do when the crescent is small enough, then there is no contrast between the disc and the crescent so it cannot be seen?"

The Malaysian scientist Ilyas also investigated the magnitude of the Danjon limit. Rather than decreasing the limit to 5° as suggested by McNally, Ilyas concluded that the critical elongation should be increased to 9–10°, below which the crescent cannot be seen because its width would be too small to produce sufficient contrast above the average eye's threshold[8]. Later Ilyas increased his limit to 10·5° (refs. 9 & 10). This is in agreement with the recent guide of the Royal Greenwich Observatory[11] on the visibility of the lunar crescent which states that, "It is unlikely that the new crescent will be visible unless the elongation exceeds 10° ...".

Ilyas derived his 10·5° limit by assuming that the lowest limit of visibility of the lunar width is $W = 0{\cdot}25'$ and using the somewhat inaccurate formula of Bruin[12] for converting the elongation to width ($W = d \sin^2(a/2)$; where d is the lunar diameter). Bruin's above simplified formula neglects the fact that the width of the lunar crescent depends on the distance of the Earth from both of the Moon and Sun.

From the observational and calendrical point of view, the *explanation* of the Danjon limit is not of much interest, but the determination of its magnitude is very important because it is a reliable criterion for rejecting any claim of sighting the lunar crescent when it is less than the Danjon limit from the Sun. This does not mean, however, that by the elongation of the Moon merely exceeding the Danjon limit the crescent would *necessarily* be visible. (For instance, Ilyas[13] refers to such a case of calendrical misuse of the Danjon limit.) Other factors, such as the altitude of the Moon and atmospheric conditions, may render the lunar crescent invisible despite having an elongation greater than the Danjon limit.

Danjon's setting of the lowest limit to 7° was not the result of direct measurement of a crescent of just above this elongation but merely the result of the extrapolation of the curve that he fitted to his data. In fact, the smallest elongation in Danjon's original data was 8°, for which the arc of deficiency was 6·2° (ref. 3, p. 60). In other words, Danjon could have only guessed at the arc of deficiency for elongations less than 8° and the value of elongation which would have the same value of the arc of deficiency. The line that he fitted to the data could have been shifted in any direction.

In order to determine the value of Danjon's limit, we have examined the Danjon limit empirically using a large number of ancient and modern observations of first and last visibility of the lunar crescent. The modern data that we used have all been made by experienced observers and have carefully been checked and published by Schaefer[14,15] and Doggett & Schaefer[7] who compiled them from the published astronomical literature as well as from Moonwatches that they organized[5]. The total number of the observations compiled by Schaefer and Doggett is 295; 271 observations of first visibility of the new Moon, 23 observations of last visibility of the old Moon, and one observation that was later proved to be false and has, therefore, been discarded in this study. One very important aspect of these 294 modern observations is that they are not all positive sightings but 88 of them are negative observations, *i.e.*, unsuccessful attempts to spot the Moon. Such negative observations are of exceptional importance in determining the limits of visibility. Additionally, these data are from various localities in the northern and southern hemispheres and their dates are in the range 1859 to 1996.

We added to the modern data observations from ancient Babylon. The Babylonians were regular watchers of the sky and they were especially interested in lunar phenomena, including the evening when the crescent would become first visible as this would indicate the beginning of the month of their luni-solar calendar. These observations were made by professional astronomers, who were also astrologers, who recorded them on clay tablets. We extracted from the recently published translations and transliterations of the Babylonian *Astronomical Diaries*[16,17,18] all entries that stated explicitly that the month was begun after actual sighting of the Moon. We neglected all entries that did not include explicit reference to actual sighting of the Moon, as the Babylonians were able to predict lunar phenomena and they used their knowledge to determine the beginning of the month when visibility of the lunar crescent was prevented by unfavorable weather. Thus, the collection that we made consists of actual sightings of the lunar crescent and not merely predictions of when it might be seen.

The Babylonian observations are from the period 568 B.C. to 74 B.C. We made the necessary checks to determine the exact Julian dates of these observations of first visibility of the lunar crescent which are dated using the Babylonian luni-solar calendar[19]. We found 209 positive observations but, unfortunately, we could not determine with confidence negative observations because the Babylonians would not state explicitly when the crescent was looked for but not seen. They would only make a mention when the Moon was not seen because of unfavourable weather, but this does not tell us whether the specific Moon was intrinsically invisible or not. Adding these observations to the recent data, the total number of ancient and modern observations used in this study is 503, 88 of which are negative sightings.

We have calculated the elongation of the Moon from the Sun at sunset for the evening observations and at sunrise for the morning observations. We have extracted from the data all the observations with elongation less than or equal to 9·4° and have included them in ascending order in Table I. The 9·4° limit was chosen as the data collection shows that naked eye observations of lunar crescents with elongations above 9·4° are commonplace, so they are of little relevance to the study of the visibility limit of elongation.

The table shows that the minimum angular distance between a visible crescent and the Sun is 7·5°, which is that of the lunar crescent of 1990 February 25 observed from 83·5° W 35·6° N. The table reports three attempts to observe that crescent from the same site, the three of which included the use of binoculars or telescope. Observer 11 succeeded in spotting the young Moon by both instrument and unaided eye; observer 12 saw the crescent only by the optical instrument and could not see it by the naked eye; whereas observer 13 failed to spot the crescent by both means.

The fact that three simultaneous observations of the same crescent produced three different results, as well as the fact that these three observations were made from a place that is as high as 1524 metres above sea level, strongly indicate that a lunar crescent of about 7·5° elongation is very difficult to spot, though not necessarily impossible. In other words, 7·5° seems to be the lowest visibility limit of elongation. This conclusion is further supported by the fact that observer 13 failed to spot the crescent even with the aid of an optical instrument. Table I shows that all of the ten lunar crescents with elongations less than 7·5° were not seen. The 5° limit suggested by McNally[4] is certainly too low. The same can be said of the 7° limit suggested by Danjon[1,3] and accepted by Schaefer[6].

On the other hand, the 10·5° limit suggested by Ilyas[9] is an underestimation of the visibility limit of the human eye. In fact, our full set of data includes 28 positive naked-eye observations of lunar crescents with elongation less than 10·5°. In fact, even the recent guideline[11] of the RGO that "It is unlikely that the new crescent will be visible unless the elongation exceeds 10°" is an overestimation. The set of data that we compiled includes 20 successful naked-eye observations of crescents less than 10° from the Sun.

It might well be the case, of course, that the shortening in the length of a crescent that is 7° from the Sun is such that no part of it remains lit, but then this does not mean necessarily that the crescent that is just above 7° of elongation would be visible, as concluded by Danjon. Therefore, according to the available observational data, 7·5° is the smallest visible elongation and the invisibility limit of elongation could be just below that.

Table I also tells us that, apart from the crescents of observations 11 and 35 (both of which were detected at high altitudes), all the crescents with elongation in the range 7·5–8·9° were missed by the unaided eye. Of these 27 crescents that escaped sighting by the naked eye, 19 were tried with the help of binoculars or telescope, but still 6 of the 19 evaded detection. Apart from observations 11 and 35, the crescent with the second smallest elongation that has

TABLE I

The Entries with Elongation $\leqslant 9{\cdot}4^\circ$

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1	222	1984	1	3	E	−35·6	15·6	335	I		4·8
2	103	1922	4	27	E	−18·5	−33·9	30·5	I		5·2
3	242	1989	6	3	E	155·5	19·8	4255	I	I	6·1
4	3	1860	1	23	E	−23·7	38·0	122	I		6·2
5	41	1871	6	18	E	−23·7	38·0	122	I		6·3
6	231	1988	4	16	E	84·1	37·2	305	I	I	6·6
7	232	1988	5	15	M	84·1	37·2	305	I	I	6·7
8	227	1987	4	27	M	84·1	37·2	305	I	I	7·0
9	288	1992	4	2	M	155·5	19·8	4176	I	I	7·1
10	271	1984	9	25	E	−35·6	15·6	335	I		7·3
11	278	1990	2	25	E	83·5	35·6	1524	V	V	7·5
12	279	1990	2	25	E	83·5	35·6	1524	I	V	7·5
13	280	1990	2	25	E	83·5	35·6	1524	I	I	7·5
14	292	1996	1	20	E	111·0	32·4	853	I	V	7·6
15	293	1996	1	20	E	113·2	32·8	259	I	V	7·6
16	226	1985	4	20	E	84·1	37·2	305	I	I	7·8
17	294	1996	1	20	E	118·3	34·1	530	I	V	7·8
18	295	1996	1	20	E	118·3	34·1	530	I	I	7·8
19	252	1990	5	24	E	83·5	35·6	1524	I	V	7·8
20	258	1984	2	2	E	−35·6	15·6	335	I		7·8
21	15	1862	4	29	E	−23·7	38·0	122	I		7·8
22	289	1995	1	1	E	106·0	33·0	1219	I	V	7·9
23	239	1989	5	5	E	84·8	42·7	259	I	V	8·1
24	240	1989	5	5	E	84·8	42·7	259	I	I	8·1
25	237	1989	5	5	E	85·7	43·0	244	I	V	8·1
26	20	1864	5	6	E	−26·2	39·6	122	I		8·1
27	256	1984	1	3	E	84·1	37·2	305	I	I	8·2
28	94	1921	2	8	E	6·2	36·5	0	I		8·2
29	234	1988	6	14	E	84·1	37·2	305	I	I	8·3
30	95	1921	2	8	E	9·1	38·8	0	I		8·3
31	253	1983	11	5	E	−35·6	15·6	335	I		8·3
32	241	1989	5	5	E	97·0	30·3	183	I	V	8·3
33	275	1984	11	23	E	−35·6	15·6	335	I		8·3
34	120	1972	3	15	E	117·6	35·5	914	I	B	8·6
35	119	1972	3	15	E	117·6	35·5	1128	V	B	8·6
36	228	1987	6	26	E	71·0	−30·1	2774	I	B	8·6
37	281	1990	5	24	E	110·5	31·6	1372	I	V	8·7
38	238	1989	5	5	E	105·5	39·7	3353	I	V	8·7
39	98	1921	10	31	E	−18·5	−33·9	30·5	I		8·9
40	250	1990	5	24	E	118·1	34·2	530	I	V	9·0
41	251	1990	5	24	E	118·1	34·2	530	V	V	9·0
42	86	1913	11	28	E	−18·5	−33·9	91·4	V		9·1
43	166	1981	7	30	M	71·3	42·3	30	I		9·1
44	52	1873	4	27	E	−23·7	38·0	122	I		9·2
45	158	1979	1	28	E	81·3	29·9	0	V		9·2
46	106	1931	8	13	M	−7·7	48·6	122	I	T	9·3
47	159	1979	1	28	E	82·4	29·7	0	V	B	9·3
48	194	1987	6	25	M	71	−30·1	2774	I	B	9·3
49	224	1984	5	1	E	84·1	37·2	305	I	I	9·3
50	201	1987	6	26	E	84·1	37·2	305	I	V	9·3
51	157	1979	1	27	M	111·7	35·2	2469	V	B	9·4
52	161	1979	1	28	E	90·3	38·7	183	V	B	9·4

The columns contain the following information: (1) reference number, (2) reference number given by Schaefer and Doggett for the modern observations, (3–5) date, (6) the time of the observation (E = evening, M = morning), (7–9) coordinates and altitude of the observing site (in metres), (10) the result of observing with the unaided eye (I = invisible, V = visible), (11) the result of observing with an optical instrument (I = invisible, B = visible by binoculars, T = visible by telescope, V = visible but the original source does not specify whether binoculars or telescope were used) (a blank indicates that no optical aid was used), and finally (12) the elongation in degrees (allowing for parallax).

been seen by the unaided eye in Table 1 is 9° of observation 41. But this was also spotted with binoculars in addition to the naked eye, and it was sighted from a place which is 530 metres above sea level. The crescent with the smallest elongation that has been seen by the unaided eye and whose detection did not include the use of optical help nor watching from a high place is that of observation 42 which was 9·1° away from the Sun at sunset. The complete set of the 503 observations shows that the minimum visible elongation in the Babylonian data is 9·8°. As already mentioned, the original data shows that modern naked-eye observations of lunar crescents with elongations above 9·4° are commonplace.

From the above discussion, it seems reasonable to conclude that 7·5° is the lowest visibility limit of elongation (*i.e.*, the limit below which the crescent would be invisible because of the phenomenon of shortening of the outer terminator, regardless of the availability of whatever favourable visibility conditions). However, there is little chance that the crescent would be seen when it is less than 9° away from the Sun at sunset. Therefore, the 5° limit of McNally[4] and the 7° limit suggested by Danjon and accepted by Schaefer[6] seem to be underestimations, whereas the 10° and 10·5° limits of the RGO[11] and Ilyas[9] are both overestimated.

Acknowledgement

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